Mass Sum Rules for Vector Mesons and Properties of ω and φ Mesons*

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A mass sum rule for nonet vector mesons and decay and production properties of ω and φ are investigated, assuming a particular broken SU(3) symmetry in strong interactions. A simple model in which boson states are assumed to be composed of two SU(3) triplets is considered in detail, and the following mass sum rule for the nonet is derived: $2M_{K^*}-M_\varphi \approx M_\rho-(M_\omega-M_\rho)/2$. A mixing angle between the $T=0$ octet partner and the singlet can be fixed by the observed $\omega-\rho$ mass difference, which gives $\cos\theta\approx 0.64$. It is suggested that the decay rate of $\varphi\to\rho+\pi$ and the $g_{\varphi NN}$ coupling constant is proportional to the $\omega-\rho$ mass difference, in which case the two coupling constants g_B and g_Y, corresponding to baryon and hypercharge currents, respectively, can be fixed as follows: $g_B{}^2/4\pi\approx 3$, $g_Y{}^2/4\pi\approx 1.5$. This model also suggests a small F versus D ratio for 0^- octet-mesons and $P_{1/2}$ octet-baryon couplings.

1. INTRODUCTION—A SIMPLE MODEL

THE most remarkable aspects required of a broken SU(3) symmetry model for strong interactions are firstly, classifications of the many strongly interacting particles and resonances into supermultiplets predicted by the SU(3) group, and secondly, the successful applications of Gell-Mann–Okubo mass sum rules among supermultiplets.

We now have impressive experimental evidence confirming the classification of $P_{1/2}$ octet and $P_{3/2}$ decuplet baryons and 0^- octet mesons and also the linear mass [or (mass)^2] sum rules among members of each of these representations. For 1^- mesons, however, we observe nine vector mesons instead of eight. They are usually described in terms of an octet and a singlet with possible mixture of the $T=0$ octet partner ω_0 and the unitary singlet φ_0. A straightforward application of the mass rules determines a mixing angle θ which is found to be $\cos\theta\approx 0.64$.[1] The observed ω and φ mesons are, therefore, almost equal mixtures of ω_0 and φ_0 mesons.

However, when we look at their experimental behavior, these two mesons have quite different properties: (i) the decay width of ω into 3π, $\Gamma(\omega\to 3\pi)$ is about 9 MeV,[2] while the corresponding width of φ is very small,[3] perhaps less than 1 MeV. (ii) ω is copiously produced in both πN and $N\bar{N}$ reactions while φ is not.[4]

In a previous paper[5] the author attempted to explain these properties in terms of R invariance without assuming SU(3) symmetry. But, since we now have convincing evidence for approximate SU(3) symmetry in strong interactions, we find it desirable to treat these nine mesons in a more unified way based on SU(3) model, taking into account the properties of ω and φ stated above. However, if R invariance is simply incorporated into SU(3) symmetry scheme, we immediately find the following troubles: a vanishing neutron magnetic moment, existence of $\overline{10}$ baryon multiplets, no coexistence of F- and D-type couplings between octet 0^- mesons and octet $P_{1/2}$ baryons, and so on. Therefore, in this paper, instead of assuming R invariance, we try to construct a simple SU(3) symmetry model for 1^- nonet mesons which equally well accommodate the observed properties of ω and φ.

Okubo was the first to attempt to treat nine vector mesons as a nonet and derived a mass rule among the nonet.[1] The same mass rule was also derived by Gürsey, Lee, and Nauenberg from a somewhat different point of view.[6]

It is theoretically appealing to assume unitary triplets as fundamental objects and to describe the observed multiplets as composite states of these triplets. Historically, the symmetrical Sakata model[7] assumed one unitary triplet and identified this triplet as p, n, and Λ. But we can treat these triplets in a more abstract manner such as Gell-Mann's quarks scheme.[8] Actually, Gürsey, Lee, and Nauenberg assumed the nonet 1^- mesons to be composite states of a triplet β with $\frac{1}{2}$ integer spin and integer charge and derived the mass rule, assuming the usual form for the symmetry breaking interaction. But, in order to guarantee a degeneracy between octet and singlet states generated by $\bar{\beta}\beta$, they have to impose some restrictions on the interaction

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[6] F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. **135**, B467 (1964).

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[8] M. Gell-Mann, Phys. Letters **8**, 214 (1964).

Hamiltonian which, if strictly applied within the framework of local field theory, leads to violations of crossing symmetry and the asymptotic conditions. It will be noted also from their mass formula that ω and ρ mesons remain degenerate.[9]

In order to remove these difficulties, we assume two independent SU(3) triplets, x and y. Vector mesons are assumed to be composite states of these two triplets, like $\bar{x}y$ or $x\bar{y}$. Symmetry properties are described by two independent SU(3) transformation groups which are associated with the x and y fields, respectively.[10] Vector meson states generated by $\bar{x}y$ and their adjoint $x\bar{y}$ necessarily form a nonet since the two SU(3)'s are independent.[11] Octet 0^- mesons (and possibly singlet 0^- mesons) may be composed of $\bar{x}x$ or $\bar{y}y$ or linear combinations of these two.[12]

Within this model, it is interesting to speculate that strong interactions are classified into two classes; the primary interaction and the secondary one. The primary interaction is assumed to be invariant with respect to two independent SU(3)[x] and SU(3)[y] transformations. On the other hand, the secondary interaction contains two terms; one is invariant under simultaneous SU(3) transformations, and the other is assumed to transform according to the $T_3{}^3$ component of the generators $T_l{}^k$ of the total system, i.e., $T_3{}^3[x]+T_3{}^3[y]$. Further, we assume there exist strong attractive forces among $\bar{x}x$, $\bar{y}y$, and $\bar{x}y$, but repulsive forces among xx, yy, and xy which produce supermultiplets consistent with the observations. Nonet vector mesons are, in this scheme, degenerate under the primary interaction while mass splittings among the nonet members are induced by the secondary interaction.

The mass sum rule for such 1^- nonet mesons is derived following Okubo's method,[13] which gives the following result (Sec. 2),

$$2M_{K^*}-M_\varphi \approx M_\rho - \tfrac{1}{2}(M_\omega - M_\rho).$$

Note that ω and ρ are no longer degenerate as in Okubo, Gürsey, Lee, and Nauenberg mass formula—hereafter abbreviated as the OGLN formula. The mixing angle θ can be determined by making use of the observed mass difference between ω and ρ, which gives just $\cos\theta \approx 0.64$.

In Sec. 3, we incorporate decay and production properties of ω and φ into this scheme, which, supplemented with an additional hypothesis that nonet vector mesons are associated with conserved currents, leads to the following results: (i) We can give a plausible argument for a suppression of the $\varphi \to \rho + \pi$ decay mode. (ii) Two coupling constants, g_B and g_Y, corresponding to symmetric and antisymmetric octet currents of baryons, are almost fixed by a requirement that $g_{\varphi NN}$ coupling should be minimized. It is suggested then that the $g_{\varphi NN}$ coupling constant is proportional to the $\omega-\rho$ mass difference and is therefore small. (iii) It turns out that φ strongly couples to cascade particles, $g_{\varphi\Xi\Xi}{}^2 \gg g_{\varphi NN}{}^2$. (iv) On the other hand, ω strongly couples to nucleons, while weakly to cascade particles, $g_{\omega NN}{}^2 \gg g_{\omega\Xi\Xi}{}^2$.

Finally, in Sec. 4, we give several remarks about our model and discuss possible extensions to baryon multiplets.

2. MASS SUM RULE FOR VECTOR MESONS

Following Okubo,[1] we describe nonet 1^- mesons by a self-adjoint tensor $F_l{}^k$ expressed as

$$F_l{}^k = f_l{}^k + (1/\sqrt{3})\delta_l{}^k \varphi_0, \tag{2.1}$$

where $f_l{}^k$ stands for the usual octet 1^- mesons and φ_0 is a singlet 1^- meson.

The mass sum rule for these vector mesons are easily obtained under the assumption of a particular transformation property for the symmetry breaking interaction. We have, in lowest order of this symmetry breaking interaction, the following expression for the expectation value of $T_3{}^3$,

$$\langle T_3{}^3\rangle = A_0 F_l{}^k F_k{}^l + A_1 F_k{}^k F_l{}^l + A_2 F_l{}^3 F_3{}^l. \tag{2.2}$$

It will be noted that a term like $F_3{}^3 F_l{}^l$ does not appear in Eq. (2.2).[14]

From Eq. (2.2), we obtain[15]

$$\begin{aligned}
M_\rho &= m_0,\\
M_{\omega_0} &= m_0 + \tfrac{4}{3}m_1,\\
M_{\varphi_0} &= m_0 + \tfrac{2}{3}m_1 + 6m_2,\\
M_{K^*} &= m_0 + m_1,\\
M_{\omega_0-\varphi_0} &= -\tfrac{2}{3}\sqrt{2}m_1,
\end{aligned} \tag{2.3}$$

where $M_{\omega_0-\varphi_0}$ is the transition mass between ω_0 and φ_0. The mass matrix can be diagonalized by putting

$$\begin{aligned}
\omega &= \cos\theta\omega_0 + \sin\theta\varphi_0,\\
\varphi &= -\sin\theta\omega_0 + \cos\theta\varphi_0,
\end{aligned} \tag{2.4}$$

[9] Recently, Singer attempted to remove the degeneracy of ω and ρ by means of electromagnetic self-energy; P. Singer, Phys. Rev. Letters **12**, 524 (1964).

[10] Recently the symmetry of $SU_3 \times SU_3$ has been discussed by several authors. See J. Schwinger, Phys. Rev. Letters **12**, 237 (1964). R. E. Cutkosky, *ibid.* **12**, 530 (1964). I. S. Gerstein and K. T. Mahanthappa, *ibid.* **12**, 570 (1964).

[11] Since vector nonet is described by a self-adjoint tensor $F_l{}^k$ with a property $C^{-1}F_l{}^kC = -F_k{}^l$, we must identify observed vector mesons with symmetric or antisymmetric combinations of $(\bar{x}y)$ and $(\bar{y}x)$ according to the assumption that $(\bar{x}y)$ is transformed to $-(\bar{y}x)$ or $+(\bar{y}x)$ under the operation of charge conjugation. See also discussions in Sec. 4.

[12] In this paper, we do not pursue such problems as how to extend the two triplets model to accommodate $P_{1/2}$ octet and $P_{3/2}$ decuplet baryons seriously. There are many varieties of way for constructing unified but schematic models by assuming several SU_3 triplets (Ref. 6). We assume, in this paper, these baryons supermultiplets in completely *ad hoc* manner.

[13] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

[14] This is because we assumed that the symmetry breaking interaction is transformed according to $T_3{}^3[x]+T_3{}^3[y]$. Clearly, $F_3{}^3$ which is transformed as $\bar{x}^3y_3$ or $\bar{y}^3x_3$, can not have a correct transformation property.

[15] We assume linear mass formula for the nonet without any theoretical justifications. See, however, Ref. 6.

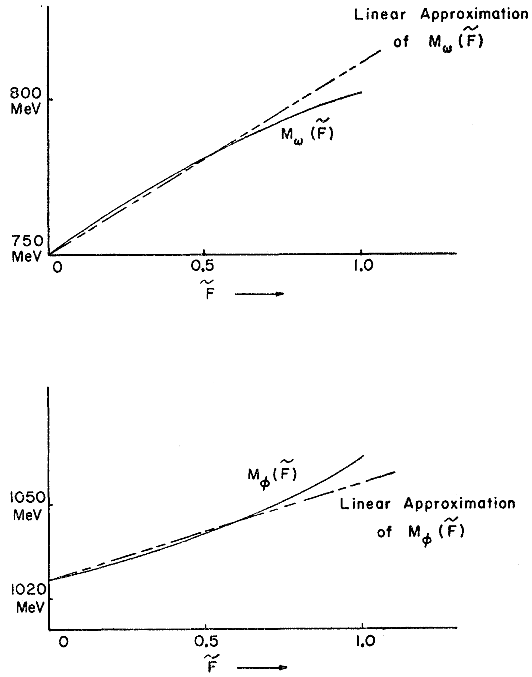

FIG. 1. Masses of ω and φ as a function of $\tilde{f}$ are shown as solid lines, and the corresponding curves of linear approximations as dotted lines. We have used the mass values $m_0=750$ MeV, $m_1=92$ MeV so as to reproduce the observed masses of M_ρ and M_{K^*}.

with

$$\cos^2\theta=[(8+f^2)^{1/2}-f]/2(8+f^2)^{1/2}, \qquad (2.5)$$

where

$$f=(m_1-m_2)/m_1. \qquad (2.6)$$

The physical masses for the vector mesons are given by

$$M_\rho=m_0,$$
$$M_\omega=m_0+[2\cos^2\theta+(2-f)\sin^2\theta -2\sqrt{2}\cos\theta\sin\theta]m_1, \qquad (2.7)$$
$$M_\varphi=m_0+[2\sin^2\theta+(2-f)\cos^2\theta +2\sqrt{2}\cos\theta\sin\theta]m_1,$$
$$M_{K^*}=m_0+\tfrac{3}{2}m_1,$$

with θ and f defined in Eqs. (2.5) and (2.6).

We note that in the particular case of $m_2=0$ (or $f=1$), the mixing angle is uniquely given by $\cos\theta=\frac{1}{3}\sqrt{3}$ and we just get the OGLN mass formula. The masses of ω and ρ are degenerate in this approximation.

Next, instead of setting $f=1$, we assume $f\approx 1$ and obtain from Eq. (2.5),

$$\cos^2\theta\approx\tfrac{1}{3}\{1+4(1-f)/9\}, \qquad (2.8)$$

together with the following linear mass sum rule for the nonet,

$$2M_{K^*}-M_\varphi\approx M_\rho-\tfrac{1}{2}(M_\omega-M_\rho), \qquad (2.9)$$

which is consistent with the observed masses within experimental errors.

We plot, in Fig. 1, the ω and φ meson mass shifts as a function of the parameter f. It will be noted that the linear approximations (2.8) and (2.9) are good so long as f is not very small.

The value of f can be estimated by making use of the observed mass difference between ω and ρ, which gives

$$f\approx\tfrac{1}{2}. \qquad (2.10)$$

This will be used in the next section as a tentative value for the parameter f to estimate decay and production ratios of ω and φ.

3. DECAY AND PRODUCTION OF ω AND φ MESONS

In this section, we attempt a phenomenological analysis for decay and production properties of ω and φ mesons from the point of view that they are members of a nonet with the mixing parameter given by (2.8) and (2.10). The symmetry breaking interactions are neglected.

3.1 Decay Rate of $\varphi\to K+\bar{K}$

The decay process of 1^- nonet into two 0^- octets is described by the effective interaction

$$H=(ig/\sqrt{2})F_{l\alpha}{}^k\{M_m{}^l\partial_\alpha M_k{}^m-\partial_\alpha M_m{}^l M_k{}^m\}, \qquad (3.1)$$

where $M_l{}^k$ represents the 0^- octet.

The decay of $\varphi\to K+\bar{K}$, which proceeds through the ω_0 component of φ, can be easily computed from (3.1), with the result

$$\Gamma(\varphi\to K+\bar{K})\approx(g^2/4\pi)(1-\tfrac{1}{9}\tilde{f})^2(k^3/3M_\varphi{}^2), \qquad (3.2)$$
$$\approx 2\text{ MeV}. \qquad (3.3)$$

We have used the values of[16] $\tilde{f}\approx\frac{1}{4}$ and $g^2/4\pi\approx 2$, the latter being determined from the data on the ρ meson width, $\Gamma(\rho\to 2\pi)\approx 100$ MeV.

3.2 Decay Rate of $\varphi\to\rho+\pi$

The apparent absence of the $\varphi\to\rho+\pi$ decay mode, which otherwise is expected to compete quite favorably with the $\varphi\to K+\bar{K}$ mode, is one of the most puzzling properties of the φ meson.

According to our scheme, the effective interaction Hamiltonian invariant under SU(3) is given by

$$H=i\gamma_0\mathcal{E}_{\mu\nu\lambda\sigma}\partial_\mu F_{l\nu}{}^k\partial_\lambda F_{m\sigma}{}^l M_k{}^m +i\gamma_1\mathcal{E}_{\mu\nu\lambda\sigma}\partial_\mu F_{k\nu}{}^k\partial_\lambda F_{m\sigma}{}^l M_l{}^m. \qquad (3.4)$$

It is interesting to note that if we set the second term of (3.4) equal to zero, the ratio of the two coupling constants $\gamma_{\varphi\rho\pi}$ and $\gamma_{\omega\rho\pi}$ can be determined from Eq. (3.4) and expressed in terms of f as follows:

$$\gamma_{\varphi\rho\pi}{}^2/\gamma_{\omega\rho\pi}{}^2\approx(2/81)(\tilde{f}^2)/[1+(2/9)\tilde{f}]^2, \qquad (3.5)$$
$$\approx 1/180. \qquad (3.6)$$

[16] $f+\tilde{f}=1$.

From this result, we obtain the following ratio for ω and φ into 3π,

$$\frac{\Gamma(\varphi\rightarrow 3\pi)}{\Gamma(\omega\rightarrow 3\pi)}\approx 1/10\sim 1/20, \tag{3.7}$$

or, by making use of the experimental widths $\Gamma(\omega\rightarrow 3\pi)\approx 9$ MeV and $\Gamma(\varphi\rightarrow K+\bar{K})\approx 3$ MeV, we have

$$\Gamma(\varphi\rightarrow 3\pi)\approx(0.5\sim 1)\text{MeV} \tag{3.8}$$

and

$$\Gamma(\varphi\rightarrow 3\pi)/\Gamma(\varphi\rightarrow K+\bar{K})\approx 0.2\sim 0.3. \tag{3.9}$$

These results are not inconsistent with the presently existing experiments.

Of course, we have no convincing arguments for setting γ_1 equal to zero. But, we would like to remark that this second term can not be invariant under a SU(3)[x] $\times$SU(3)[y] transformation. Therefore, it is quite conceivable that the first term is dominant over the second one, thus leading to a considerable suppression of the $\varphi\rightarrow\rho+\pi$ decay mode compared to the $\omega\rightarrow\rho+\pi$ one.

3.3 Conserved Currents

Let us consider the coupling scheme between $P_{1/2}$ octet baryons and 1^- nonet mesons. In this case, with the assumption that the nonet is associated with the conserved current, we have the following interaction Hamiltonian

$$\begin{aligned}H=&(ig_B/\sqrt{3})F_{k\alpha}{}^k(\bar{B}_m{}^l\gamma_\alpha B_l{}^m)\\&+(ig_Y/\sqrt{3})F_{l\alpha}{}^k(\bar{B}_m{}^l\gamma_\alpha B_k{}^m-\bar{B}_k{}^m\gamma_\alpha B_m{}^l),\end{aligned} \tag{3.10}$$

where the $P_{1/2}$ octet baryons are denoted by $B_l{}^k$.

The first term corresponds to the baryon current interaction and the second to the F-type interaction which is antisymmetric under the operation R. Obviously, ω_0 and φ_0 are associated with hypercharge and baryon currents, respectively.[17]

First, let us consider again the particular limit of $M_\omega=M_\rho$ (or $f=1$). It can be shown, then, that φ is completely decoupled to the nucleon by requiring a relation, $g_B=\sqrt{2}g_Y$. The relevant interaction Hamiltonian in this case is explicitly written down as (vector notations are suppressed)[18]

$$\begin{aligned}&(ig_Y/\sqrt{3})[3\bar{N}N+\bar{\Xi}\Xi+2\bar{\Lambda}\Lambda+2\bar{\Sigma}\Sigma]\omega\\&\quad+(ig_Y/\sqrt{3})[-2\sqrt{2}\bar{\Xi}\Xi-\sqrt{2}\bar{\Lambda}\Lambda-\sqrt{2}\bar{\Sigma}\Sigma]\varphi.\end{aligned} \tag{3.11}$$

Note that with this particular relation between the coupling constants g_B and g_Y, φ is strongly coupled to the cascade particle, while ω is strongly coupled to the nucleon but weakly to cascade particle. The coupling constant g_Y may be estimated from the conserved current hypothesis and observed ρ meson width, yielding the well-known result,

$$g_Y{}^2/4\pi\approx\tfrac{3}{4}g_{\rho\pi\pi}{}^2/4\pi\approx 1.5. \tag{3.12}$$

For the purpose of numerical orientation, we list various values of the coupling constants which are computed from (3.11) and (3.12),

$$\begin{aligned}&g_{\omega NN}{}^2/4\pi\approx 4.5,\quad g_{\varphi NN}{}^2/4\pi\approx 0,\\&g_{\omega\Xi\Xi}{}^2/4\pi\approx 0.5,\quad g_{\varphi\Xi\Xi}{}^2/4\pi\approx 4.0,\\&g_{\omega KK}{}^2/4\pi\approx 0.5,\quad g_{\varphi KK}{}^2/4\pi\approx 1,\end{aligned} \tag{3.13}$$

and

$$g_{\rho NN}{}^2/4\pi\approx g_{\rho\Xi\Xi}{}^2\approx 0.5\quad g_{\rho\pi\pi}{}^2/4\pi\approx 2.$$

Inclusion of the mass difference effect for ω and ρ does not change the essential features of the coupling scheme presented here so far as we keep the relation $g_B=\sqrt{2}g_Y$. It is true that φ is no longer completely decoupled to the nucleon if we include the $\omega-\rho$ mass difference but the ratio of the two coupling constants $g_{\varphi NN}$ and $g_{\omega NN}$, for instance, is given by

$$\begin{aligned}g_{\varphi NN}{}^2/g_{\omega NN}{}^2&\approx(2/81)\bar{f}^2\\&\approx 10^{-2}\end{aligned} \tag{3.14}$$

which is very small.

In view of the results (3.5) and (3.14), one may expect that the production ratios of ω and φ in both πN collision and $N\bar{N}$ annihilation processes are approximately given by

$$\begin{aligned}&\sigma(\pi+N\rightarrow\varphi+N+n\pi)/\sigma(\pi+N\rightarrow\omega+N+n\pi),\\&\approx\sigma(N+\bar{N}\rightarrow\varphi+n\pi)/\sigma(N+\bar{N}\rightarrow\omega+n\pi)\approx 10^{-2},\end{aligned} \tag{3.15}$$

which appear not to be inconsistent with observations.[4]

Before closing this subsection, a remark is in order. Since, in our model, φ is rather strongly coupled to Σ and Ξ, the decay interaction effective in the $\varphi\rightarrow\rho+\pi$ mode will be induced through the following two-step processes: $\varphi\rightarrow(\Sigma+\bar{\Sigma}$ and $\Xi+\bar{\Xi})\rightarrow\rho+\pi$, which should be canceled, leading to a great suppression for this decay mode. Within a simple perturbation calculation, this implies

$$\gamma_{\varphi\rho\pi}{}^2\text{ induced}\propto(g_{\varphi\Xi\Xi}g_{\pi\Xi\Xi}+2g_{\varphi\Sigma\Sigma}g_{\pi\Sigma\Sigma})^2, \tag{3.16}$$

$$\approx 0. \tag{3.17}$$

Equations (3.16) and (3.17) imply a condition for the ratio of F-type and D-type 0^- octet-meson and $P_{1/2}$ octet-baryon interactions.

Substituting the result (3.11) into Eq. (3.16), we find

$$F\approx 0.25,\quad D\approx 0.75, \tag{3.18}$$

which are qualitatively in agreement with those obtained by several physicists employing the bootstrap mechanism.[19]

[17] Similar model was considered by R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964).

[18] One of the motivations for setting $g_{\varphi NN}=0$ is that we observe a small production ratio of φ compared to ω not only in πN collisions but also in $\bar{N}N$ annihilations.

[19] R. E. Cutkosky, Ann. Phys. (N. Y.) **23**, 415 (1963). A. W. Martin and K. C. Wali, Nuovo Cimento **31**, 1324 (1964). Y. Hara, Phys. Rev. **133**, B1565 (1964), and R. H. Capps (to be published). In this subsection we neglect, for simplicity, the $\omega-\rho$ mass difference effect.

3.4 Radiative Decays of ω and φ

With the assumption that the electromagnetic current j_α transforms according to the $T_1{}^1$ component of a traceless tensor $T_k{}^l$ under SU(3) transformations, the neutral vector meson and the virtual photon transition is effectively described by

$$eM^2/Gf_{1\alpha}{}^1A_\alpha\,, \tag{3.19}$$

where in the limit of unitary symmetry M and G stand for vector-meson mass and the dimensionless coupling constant, respectively. The latter, according to Gell-Mann and Zachariasen,[20] is given by $G=2g_Y$ with g_Y defined by Eq. (3.4).

It is easily seen from (3.19) that the $\omega-\gamma$ and $\varphi-\gamma$ transition rates are directly related to the mixing angle θ (or f) as was pointed out by Glashow.[1] The ratio of these two coupling is given in our model by

$$G_{\varphi-\gamma}{}^2/G_{\omega-\gamma}{}^2\approx 2\,. \tag{3.20}$$

Similarly, we can show that we have the following relations for radiative decays of ρ, ω and φ:

$$G_{\omega\pi\gamma}{}^2\approx 9 \quad G_{\rho\pi\gamma}{}^2\,, \tag{3.21}$$

$$G_{\varphi\pi\gamma}{}^2\approx 0,\quad G_{\varphi\eta\gamma}{}^2\approx 8G_{\omega\pi\gamma}{}^2\,. \tag{3.22}$$

These relations were obtained by several authors.[1]

Some consequences which can be inferred from Eq. (3.20), (3.22) may be summarized as follows:

(i) For the decay of ω and φ into lepton pairs, we have the ratio[21]

$$\Gamma(\varphi\to l^+ + l^-)/\Gamma(\omega\to l^+ + l^-)\approx 2\,. \tag{3.23}$$

As was noted earlier, the ratio (3.23) is directly connected with the mixing angle. Therefore, experimental data for this ratio would clearly be of great interest.

(ii) The contribution of the φ meson to the isoscalar charge form factor of nucleon is unimportant compared to that arising from the ω meson. This is easily seen from the fact that φ is very weakly coupled to the nucleon.[22]

(iii) In high-energy π^0 photoproduction, the ω-meson exchange diagram is expected to be the most dominant one for a simple one-vector meson exchange mechnism.

(iv) Similarly, for ρ-, ω-, and φ-meson photoproducsion, ω is expected to be copiously produced compared to other two mesons.

4. REMARKS AND DISCUSSIONS

In order to derive the mass sum rule given in Sec. 2, it was essential that the expectation value of $\langle T_3{}^3\rangle$ does not contain a term like $F_3{}^3F_k{}^k$, which was rejected in our model by requiring that the mass splitting interaction should behave like the $T_3{}^3$ component of the generators $T_l{}^k=T_l{}^k[x]+T_l{}^k[y]$. Once this assumption is accepted, it does not matter how the nonet is constructed from the fundamental triplets. In this sense, the mass sum rule for the nonet as well as phenomenological analysis presented in Sec. 3, would be largely independent of the details of our model and therefore would have a more general validity.

It is amusing to note that within our model, the decay rates for the $\varphi\to\rho+\pi$ and $\varphi\to\rho+\gamma$ modes can be simultaneously related to the mass difference between ω and ρ consistent with observations. In the limit of $M_\rho=M_\omega$, we have the OGLN mass formula and the unique mixing angle between octet and singlet. The magnitude of this mixing angle, as is seen from Eq. (2.8), is changed little when the mass difference is included.

It can also be shown that the two coupling constants g_B and g_Y involved in the 1^- nonet and $P_{1/2}$ octet coupling can be fixed by the condition that the $g_{\varphi NN}$ coupling should be small, which, at the same time, makes the $g_{\omega NN}$ and $g_{\varphi\Xi\Xi}$ coupling constants large, leaving $g_{\omega\Xi\Xi}$ and $g_{\varphi NN}$ small. Numerical results presented in Sec. 3, however, should not be considered more than an order of magnitude estimation, because we have completely neglected the higher order effects of the symmetry breaking interaction. Nevertheless, we feel that in view of numerical results (3.6) and (3.13), one may expect that in both $\bar{N}N$ and πN reactions, ω is copiously produced compared to φ and the production branching ratios between them is around 10^{-2}, which is consistent with recent observations. On the other hand, in $\bar{K}N$ reactions both ω and φ may be produced with comparable order of magnitude. In particular, the following reactions will be useful to check the various couplings involving ω and φ,

$$K^- + p\to\Lambda+\omega(\mathrm{or}\,\varphi)\,, \tag{4.1}$$

$$K^- + p\to\Xi^- + K^+ + \omega(\mathrm{or}\,\varphi)\,. \tag{4.2}$$

Peripheral collisions of the first reaction (4.1) will be useful to obtain some idea of the ratio of $g_{\omega KK}$ and $g_{\varphi KK}$, which according to our model is given by $g_{\omega KK}{}^2/g_{\varphi KK}{}^2\approx\frac{1}{2}$. On the other hand, we may be able to get further information on the coupling constants $g_{\omega\Xi\Xi}$ and $g_{\varphi\Xi\Xi}$ by studying the second reaction (4.2).

We have not discussed baryon multiplets in our model. The simplest but much less elegant way of getting $P_{1/2}$ octet baryons would be to assume a singlet baryon Z, from which octet baryons are composed like $Z(\bar{x}x)$ or $Z(\bar{y}y)$. The $P_{3/2}$ decouplet baryons, then, may be derived from $P_{1/2}$ octet baryons and 0^- octet mesons by means of a bootstrap mechanism.[23]

It is natural, within our model, to speculate in the existence of a pair of octet (and possibly singlet) bosons

[20] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961)
[21] This ratio is in agreement with the result given in Ref. 17.
[22] This conclusion, which we arrived at in Ref. 5, is different from that of J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964).

[23] According to the results by Capps, such dynamical procedure of forming baryon multiplets seems to be incompatible with the consistency condition of the bootstrap mechanism. See Ref. 19.

and baryons with possibly different spin and parity, since we have introduced two kinds of unitary triplets and from them composed meson and baryon supermultiplets. One of the pairs was identified as 0^- octet meson and $P_{1/2}$ octet baryons, respectively. The other pair has not been observed so far but the $D_{3/2}$ octet and the recently observed $\omega\pi$, $K\pi\pi,\cdots$, resonances might form another pair of octets.

As was noted earlier, we could have two nonets $(\bar{x}y)\pm(\bar{y}x)$ for the vector meson supermultiplets. One of them was identified as the observed vector meson supermultiplet. The other vector nonet, if they exist, necessarily obey the same mass sum rule but have opposite charge-conjugation parity compared to the usual nonet. The question of whether such a nonet exists in nature will be checked in future experiments.

Note added in proof. By eliminating the parameter from Eqs. (2.5) and (2.7), we have the following mass formula:

$$(M_\omega-M_\rho)(M_\phi-M_\rho)=\tfrac{4}{3}(M_{K^*}-M_\rho)(M_\omega+M_\phi-2M_{K^*})$$

from which the mass formula (2.9) is obtained by a procedure of linearization.

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Spin Operators in the Kemmer Theory*

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A discussion of the various conserved spin operators in the Kemmer theory of spin-one particles is given. The algebraic properties and the interrelations of the various operators are also considered. The covariant forms of energy and spin projection operators are deduced and discussed.

INTRODUCTION

IN recent years there has been an increasing interest in the study of vector mesons. It is partly due to the realization that a theoretical understanding of the various elementary-particle interactions seems to be closely connected with vector mesons. For example, a charged vector-meson field has been postulated as a possible intermediary field in the weak interactions.[1] Again in the strong interactions it is shown for some simple models that the mass of the nucleon is entirely due to boson-fermion interaction if a vector meson is introduced as an elementary system.[2] Furthermore, in the recent Regge pole hypothesis, a vector field (massive photon) is necessary if one wants to consider the nucleons as Regge poles as was shown by Gell-Mann and Goldberger.[3] Hence, in view of the appreciable role of vector meson in the recent works, a discussion of the various spin operators in the Kemmer theory which describes a vector meson in a manifestly covariant form, may be of interest as these operators may find applications in the study of polarization and scattering phenomena.

In this paper we shall give a set of spin operators which all commute with the Hamiltonian and hence they can be used to remove the spin degeneracy remaining after having specified the energy. First a three-vector conserved spin operator obeying the usual angular-momentum commutation relations is discussed. This is useful for the discussion of polarization involving plane-wave states. Then using the method employed by Bargmann, Michel and Telegdi,[4] Good,[5] and Fradkin and Good,[6] an axial four-vector spin operator analogous to Bargmann and Wigner's generators of the little group[7] and an antisymmetric tensor operator are deduced. The algebraic properties and the connection between these three operators are worked out. Finally, the covariant forms of the energy and spin projection

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